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Fracture Toughness of Composites Reinforced with Discontinuous Fibres

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Measurements of the work of fracture of composites of polyester resin reinforced with chopped steel wires of various lengths are compared with the theory developed by Cooper. For composites containing aligned wires the results agree well with the model except where there is excessive resin cracking. The toughness of composites containing wires which are randomly distributed in the resin can be significantly greater than that of aligned composites with wires of similar length. This is probably due to the plastic shearing of wires not lying parallel or normal to the specimen axis.

1. Introduction

Cooper [1] has developed a model for the fracture toughness of composite materials based on a consideration of the work required to pull broken fibres from the matrix and taking into account the distribution of strength-reducing defects in the fibres. An important conclusion, so far as reinforcement with discontinuous fibres is concerned, is that the toughness is greatest when the length of the fibres is equal to the critical transfer length, l_c. This being so, it is apparent that maximum strength and maximum toughness cannot be achieved simultaneously, and that composites must be designed for an optimum combination of desired mechanical properties. The way this arises can be shown quite simply. Fibres shorter than l_c will be pulled from the matrix, rather than broken, when a crack passes through the composite. The energy of fracture will then largely be a combination of the work needed to debond the fibres from the matrix and the work done against friction in pulling the fibres out of the matrix. There is evidence that the former contribution may sometimes be quite significant, in glass-reinforced resins for example [2], but it is likely to be small in resin/metal fibre systems. The contribution to fracture energy from frictional pull-out work is given for a composite containing aligned fibres by the early Cottrell expression, [3]

$$w_1 = \frac{V_f \tau l^2}{12d} \text{ where } l < l_c \qquad (1)$$

per unit area of fracture surface, where d is the wire diameter, V_f is the volume fraction of fibres, τ is the dynamic interface frictional stress. Thus the work is proportional to the square of fibre length.

When the fibres are longer than l_c, a proportion l_c/l, of those bridging a matrix crack will have an end within $l_c/2$ of the plane of fracture and these will pull out; the rest will fracture [4]. Of N fibres intersecting the fracture plane, then, $N/2\,(l_c/l)$ will be left protruding from each fracture surface, and their lengths (pull-out lengths) will vary from 0 to $l_c/2$ with a mean value $l_c/4$. The frictional work of fracture of the composite is again derived from the Cottrell expression, and is now:

$$w_2 = \frac{V_f \tau l_c^3}{12d\,l} \text{ where } l > l_c \qquad (2)$$

This is inversely proportional to fibre length. If the matrix or the fibres themselves contribute a significant fracture energy, this must also be accounted for. In our experiments the fracture energy, γ_F, of the resin was known to be 200 J m^{-2} [5] and this contributes a term

$$w_3 = (1 - V_f)\,\gamma_{F(\text{resin})} \qquad (3)$$

while the fracture toughness of the wires accounts for an energy

$$w_4 = V_f(1 - l_c/l)\,\gamma_{F(\text{wires})} \text{ where } l > l_c \qquad (4)$$

since for $l > l_c$, a fraction $(1 - l_c/l)$ of the wires will be broken and not pulled out. The total

*Now at Department of Electrical Engineering, Imperial College, London, UK fracture energy of the composite, γ_F, can therefore be written:

$$\left.\begin{aligned}\gamma_F &= w_1 + w_3 \qquad \text{where } l < l_c \\ \gamma_F &= w_2 + w_3 + w_4 \text{ where } l > l_c\end{aligned}\right\} \quad (5)$$

We have tried to compare these theoretical expressions with the measured fracture energies of composites of polyester resin reinforced with aligned, chopped steel wires. The results have also been compared with the toughness of composites containing randomly oriented fibres, a system rather more relevant to the understanding of such materials as fibre-reinforced concrete.

2. Experimental Work

Most of the experiments were carried out with composites prepared from BXL polyester resin (with cobalt naphthanate accelerator and MEK peroxide catalyst in the proportion 50:1:1, by weight) containing cold-drawn, low carbon steel wire 0.15 mm in diameter, chopped to various lengths by the makers, National Standard Ltd of Stourport, Worcs. The fracture stress of the wire was initially 1.25 GN m^{-2}, but it fell to 1.2 GN m^{-2} after the low-temperature heat-treatment used in curing the resin.

Aligned composites were made by allowing wires to fall into a bar mould mounted in the magnetic field produced by a small Helmholtz pair of coils, resin and wires being gradually added until the mould was filled. Alignment was improved if the wires were agitated during the filling of the mould. Composites containing randomly arranged wires were made simply by pouring the wires into larger moulds containing resin. As a result of the mode of packing down there was a planar, two-dimensional randomness rather than a genuine three-dimensional array in these samples.

After curing overnight at room-temperature the composites were post-cured at 100° C for 24 h and cut into bars of the order of 10 × 7 mm in cross-section; all faces were then ground to remove gross defects and protruding wire ends and the samples were cut to length and notched with a diamond saw. The notch depth was one third of the specimen height and the notch root radius about 1 mm. In the random fibre composites the notches were cut normal to the plane of two-dimensional randomness.

Although an attempt had been made initially to arrive at controlled composite volume fractions of 0.10, this was found to be impossible because the longer fibres would not pack to such a high density, especially in random fibre composites, while the shortest fibres could not be prevented from packing to a much higher density. The density of each finished sample was therefore measured, after coating with PVC lacquer to prevent moisture damage, by weighing in air and in water. The mean volume fraction of every sample could therefore be obtained from the rule of mixtures for densities. Before fracture-testing, an attempt was made to sharpen the notches in all samples by lightly driving in a razor or scalpel blade or by lightly recutting the notch root with a very fine diamond saw. The difference in the test results was imperceptible.

The work of fracture of the aligned composites was determined by two different methods on samples of similar size and shape. Most tests were made with a miniature Hounsfield Charpy impact tester, using the full capacity of the machine (2.75 Joules). About half this was sufficient to break any sample. For comparison some tests were carried out in slow-bending using an Instron cross-head speed of 0.1 cm/min. The same four-point bend-testing geometry was used for both types of measurement, so that rate of loading and the possibility of damage due to impact (as opposed to high testing rate) were the only points of distinction between these two tests. However, the results indicated that there was no significant difference between the tests for this material, and the random fibre composites were therefore tested by the Charpy method only.

The work of fracture, γ_F, was computed in each case by dividing the work done in the test (either the impact energy absorbed or the area under the load-deflection curve) by twice the cross-sectional area of the sample measured at the notch. This is purely for comparison with results of the Griffith model for fracture of brittle materials and naturally runs into error when the fracture surface is non-planar. The volume fractions of these composites varied from 0.05 to 0.20, being highest for aligned, short fibres, and lowest for long random fibres. We have attempted to take account of the effect of the variation by normalising the work of fracture for each sample to $V_f = 0.10$. For aligned composites, the scatter in the measured densities and γ_F values within a batch of samples was quite small, and each test result was therefore normalised separately. The variation was naturally much greater for the random samples, and in this case the values of γ_F and V_f for between six and twelve samples were averaged separately and a mean value of 0.10 γ_F/V_f was obtained from these figures.

The fibre/resin interface behaviour was studied by embedding various lengths of wire in the resin and making pull-out tests in the conventional way [1] after the same resin curing cycle as had been used for the composites. The work of fracturing single wires was also determined empirically from separate tensile tests.

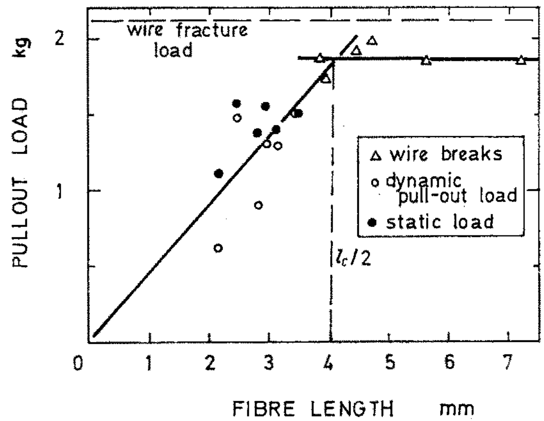

Figure 1 Results of fibre pull-out tests: low carbon steel wire in polyester resin. The interface frictional stress given by these results is $\tau = 9.5$ MN m^{-2}.

3. Results and Discussion

Results of fibre pull-out tests are shown in fig. 1. The indication of critical length given by these results is not precise, perhaps because of the brittleness of the resin, but it is probably close to 8 mm. The same explanation could also account for the fact that the wire failure level determined in the pull-out tests was rather lower than that determined in separate tensile tests. A relevant value for the fracture energy of the wire is not easily derived. The true fracture toughness is inappropriate because we are concerned with breaking an un-notched and ductile wire in tension rather than with the propagation of a pre-existing crack. We have therefore used, as an approximation, the area under a wire load/extension curve, again divided by twice the initial area. The average value obtained from three tests was 114 kJ m^{-2}. This is the usual order of magnitude for γ_F in tough metallic materials, but a value measured in this way may well be substantially greater than the work of fracture of a wire embedded in resin.

All of the quantities required to compute the total fracture energy, given by equation 5, are now known and these have been used to obtain the theoretical curve of γ_F versus fibre length shown in fig. 2. Equations 5 become,

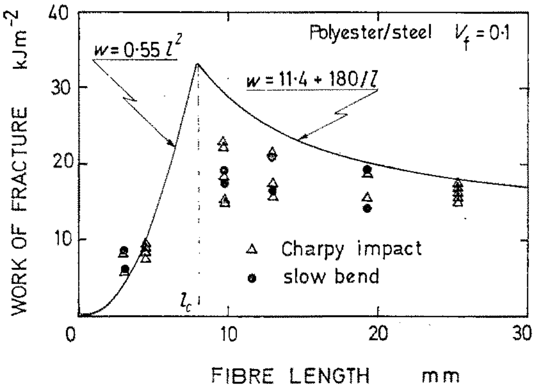

Figure 2 Work of fracture of composites containing aligned chopped steel wires as a function of fibre length. The full curve is the theoretical toughness given by the equations indicated (for l measured in mm).

$$\gamma_F = 5.5 \times 10^8\, l^2,\ \mathrm{J\,m^{-2}} \text{ where } l < l_c$$

$$\gamma_F = 11.4 \times 10^3 + 180/l,\ \mathrm{J\,m^{-2}} \text{ where } l > l_c$$

taking $V_f = 0.10$, $\tau = 9.5$ MN m^{-2}, $l_c = 8 \times 10^{-3}$ m, $d = 1.5 \times 10^{-4}$ m, $\gamma_{F(\text{wire})} = 114$ kJ m^{-2}, and ignoring the very small contribution from the resin. At the point of intersection of these two lines the predicted maximum work of fracture of the composite is 33 kJ m^{-2}. Together with the theoretical curve, fig. 2 shows all experimental points for aligned composites. The agreement is reasonably good, although the rate of fall-off of γ_F with l for $l > l_c$ is much smaller than expected and the maximum toughness obtained for 10 mm long wires is considerably less than that given by the model. The brittleness of the resin could certainly account for these discrepancies. The resin near the crack invariably fragmented during fracture, leaving a very rough fracture surface. This would mean that good contact between wires and resin was not maintained during the complete pull-out process and some frictional work would be wasted. Furthermore, since the fragmentation was more general in composites of higher volume fraction, normalisation of the data to $V_f = 0.10$ means that the results for shorter fibres have been more severely affected. *It should also be noted that the vertical position of the curve for $l > l_c$ is very sensitive to the actual value of l_c* and if, for

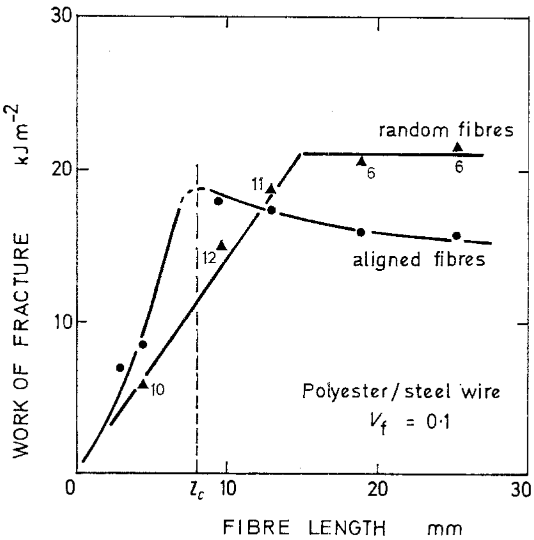

Figure 3 Fracture energy of polyester/steel composites: aligned and randomly-oriented fibres. The numbers of test samples used to obtain the averages for random composites are indicated.

example, as a result of fragmentation the effective critical length were only 7.5 mm instead of 8 mm this part of the curve would pass through data points at all fibre lengths except 10 mm.

One of the most interesting features of this work is the effect of using randomly oriented wires. The results are compared with those for aligned composites in fig. 3. They show that with the longer fibres a higher fracture toughness can be obtained at a given volume fraction if the fibre distribution is random. It appears at first sight that the form of the γ_F versus fibre length relationship has simply been modified to accommodate a higher l_c. This is incorrect, however, for inspection of the fractured surfaces showed that the mean pull-out lengths of fibres were similar in both aligned and random composites and these were in agreement with the indication from pull-out tests that the critical length was about 8 mm. The increase in toughness is most likely a result of the extra plastic work required to deform in shear those wires that are not aligned with the sample axis. Because of the random arrangement, a proportion of fibres – those oriented at or close to right angles to the specimen axis – will not cross the crack plane and will not contribute to the pull-out work. The longer the fibres are, the higher will be the proportion of the more widely misoriented ones that will intersect the crack plane however. The extra work of shearing the fibres can be crudely estimated for the model represented in fig. 4. This shows that after a matrix crack has passed an inclined wire, the wire will be straightened out by a shearing process occurring at the crack face B (the face from which the wire is being pulled out).

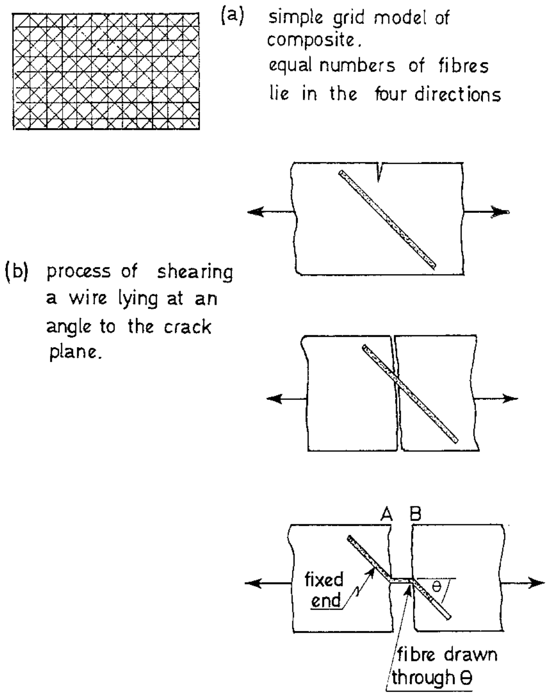

Figure 4 Model for the plastic shearing of fibres during fracture of a planar random composite.

The work of shearing per fibre is roughly $\tau_Y\,\theta$ per unit volume where τ_Y is the shear yield stress of the steel. If the mean pull-out length is $l_c/4$, and the draw angle is assumed to be $\pi/4$, we have

$$w_S = \tau_Y\,\theta(l_c/4)\,A \quad \text{J/fibre}$$

where A is the fibre cross-sectional area. If $\tau_Y = \frac{1}{2}\sigma_Y$ for the wire,

$$w_S = 6 \times 10^8\,\pi/4\,.\,A\,.\,2 \times 10^{-3}\ \text{J/fibre}$$
$$= 3.0\,\pi\,A \times 10^5\ \text{J/fibre.}$$

In an aligned, continuous-fibre composite $N(= 4V_f/\pi d^2 = V_f/A)$ fibres take part in the pull-out process. In an aligned short-fibre composite only $N/2\,.\,l_c/l$ fibres will protrude from each fracture face, and in the random model above only half of the fibres are at 45°, and are sheared in pulling out. Thus $N/4 \,.\, l_c/l$ fibres contribute to the shear work, and for 20 mm long fibres this amounts to $0.01/A$ fibres.

The work of shearing is therefore, if both fracture faces are now considered,

$$w_s = 0.01 \times 1.5\pi \times 10^5 \text{ J m}^{-2},$$

or about 5 kJ m^{-2}. It is by roughly this amount that the curve in fig. 3 for random composites lies above that for aligned samples when $l > 20$ mm. Strictly speaking this shear energy contribution will be inversely proportional to fibre length; but this is not shown in fig. 3 because the effective wire length will be limited by the specimen dimensions. This added contribution to the composite fracture energy is a significant one considering the relatively small number of fibres that is needed to produce it. It has important implications for the reinforcement of materials such as concrete, for example, where a small volume fraction of random fibres is easily incorporated but where manufacturing operations become much more complicated if larger volume fractions and aligned fibres are required. There is also the negative side of the argument, however, that randomisation of the fibres reduces considerably the strength and stiffness of the composite.

These experiments have been carried out with quite weak fibres, and the toughnesses obtained have been low in consequence. To give an indication of what could be done with high strength fibres we include a result obtained from one batch of specimens containing aligned, high-carbon steel wires, also 0.15 mm in diameter, but brass-plated because the wire stock used is manufactured for the motor tyre industry and the brass coating promotes bonding to rubber. The fracture stress of this wire was 2.54 GN m^{-2} and its fracture energy was 343 kJ m^{-2}. The critical length, l_c, is 34 mm and $\tau \approx 6$ MN m^{-2}. By the same methods as those just described, the theoretical toughness of composites with 10 vol% of 13 mm long wires is calculated to be about 60 kJ m^{-2} and the experimental mean of four test results was in fact 65 kJ m^{-2}. This is for fibres well below l_c: composites containing 34 mm long wires of the same kind should have a fracture energy over 300 kJ m^{-2} – far greater than those of conventional fibreglass and carbon-fibre-reinforced plastics materials. This high value is partly due to the high fracture strength of the wire and partly to the poor interfacial bond.

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